

Making things add up

Eric Coates explains the basic electronic circuits used in computer arithmetic



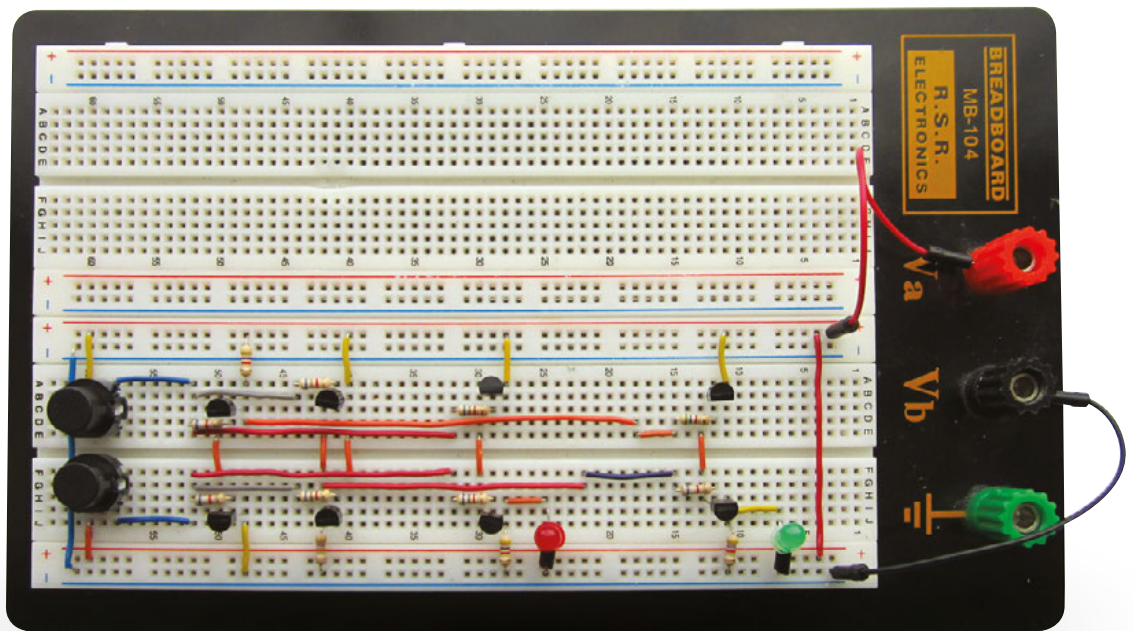
Eric Coates

hsmag.cc/cabTmd

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Right

The complete half adder circuit built on a breadboard



Computers can carry out an awesome amount of mathematics; we all know that. So how come all of this can be performed on a machine that can only add 1 and 1; no subtraction as we know it; no multiplication and no division?

The answer is that an electronic adder circuit that just adds 1 + 1 and uses a few tricks of the binary system can, with some help from a little firmware code, carry out any arithmetic we need at an amazing speed. The simple electronic calculator circuit at the heart of this wizardry is known as the Half Adder and is shown, made from just a couple of logic gates (an Exclusive OR gate and an AND gate), in **Figure 1**.

The 'half' in its name is because, while it can add 1 plus 0 and even add 0 plus 1 to get the correct answer of 1, as shown in the 'Sum' column of the truth table in Figure 1, if it adds 1 plus 1 in binary, this produces the (decimal) answer of 2, which in binary notation is 10, so a second column is needed in the form of a 'Carry' output to hold double the value of

the single-digit sum output, producing the answer of 10 (decimal 2). This is fine if we only need to add a single column of binary, but the half adder cannot cope with multi-column numbers where a carry may be produced.

THE HALF ADDER'S BIG BROTHER

Because the half adder can only add two 1-bit numbers in a single column, it does not usually work alone. However, if the carry output produced by the half adder can be used as one input to another

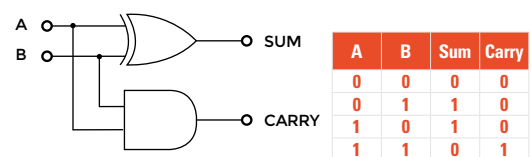


Figure 1 The half adder is at the heart of all electronic calculators

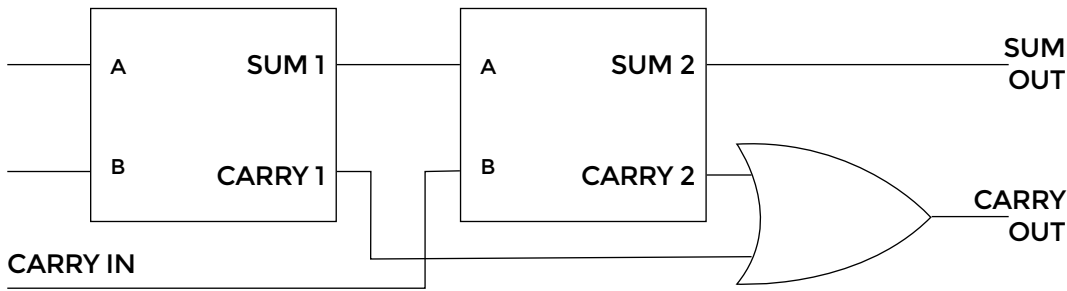


Figure 2 ♦ Two half adders make one full adder that can cope with multi-column addition

half adder, the two circuits combine to form the full adder shown in **Figure 2**, which will now have three inputs and so can add two binary bits in any one column using inputs A and B, plus any 'carry in' from a previous column, making multi-column addition possible.

So, the half adder may not carry out an impressive amount of arithmetic, but as it only takes around 30 nanoseconds to produce its answer, this basic adder could therefore carry out around 100 million similar calculations per second, which is how, when coupled with another half adder and other bits of electronic circuitry, many simple 1+1 additions are made to look like powerful maths, when it's really simple arithmetic, only at a blazing speed!

THE HALF ADDER DISSECTED

The most complex part of the half adder is an Exclusive OR (XOR) gate, which produces an output of logic 1 when its two inputs are different (01 or 10), but an output of 0 when the two inputs are the same (00 or 11). XOR gates comprise rather more electronics than the other more standard gates, which makes them slower, therefore less popular and more expensive than the other standard gates. However, the Exclusive OR logic function can be, and very often is carried out by a combination of more regular logic gates, as shown in **Figure 3** where the XOR gate is replaced by a combination of AND, OR and NAND gates, and this

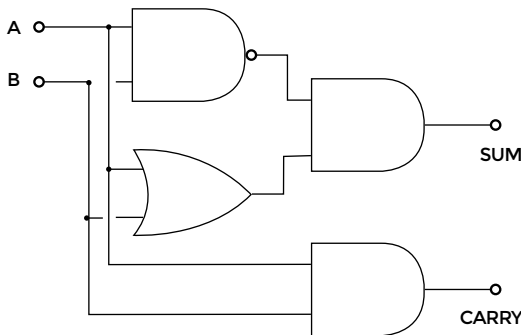


Figure 3 ♦ Equivalent half adder circuit using standard logic gates

is the approach we shall use in our transistor/resistor version of the half adder.

So why, when logic gates are readily available in integrated circuit form, and you can easily simulate their operation on a computer screen, would you want to build them from basic electronic components such as resistors and transistors? Well, these days when the Internet of Things is becoming more and more important, the skill of combining computers such as the Raspberry Pi or Arduino with external electronic circuits and devices is becoming vital. It's one thing to move things about on a computer or smartphone display, but different skills are needed to drive output devices and make things change in the real world. Therefore it's more vital than ever to understand the operation of electronic circuits at component level, and to develop the knowledge and skills needed for constructing working circuits such as the half adder.

THE AND GATE

Our half adder circuit will contain two AND gates, both working to produce the truth table in **Figure 4**. →

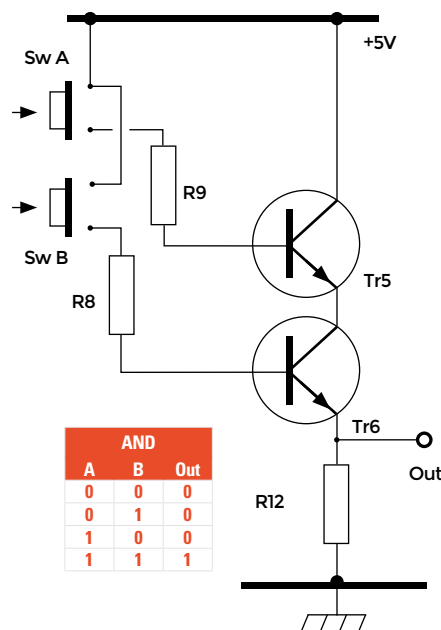


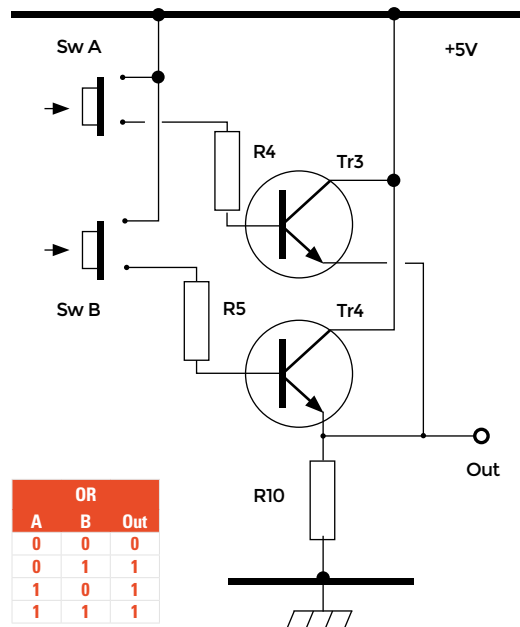
Figure 4 ♦ The AND gate output produces a logic 1 when, and only when, both inputs are at logic 1, otherwise the output is at logic 0

YOU'LL NEED

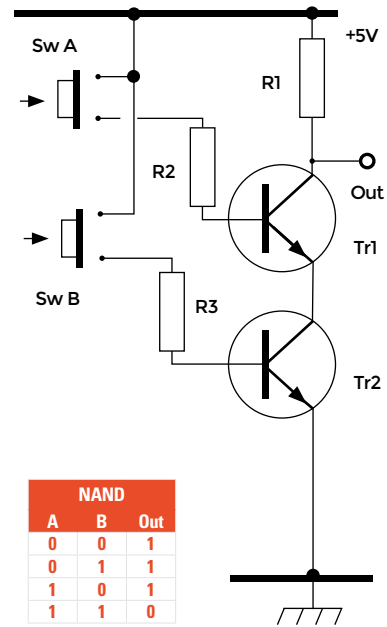
- ♦ **Transistors**
8× 2N3904
- ♦ **Resistors**
4× 4.7 kΩ (4K7)
for R1, R10, R11 & R12
- ♦ **8× 6.8 kΩ (6K8)**
for all other resistors
- ♦ **Switches**
2× Mini push to close
- ♦ **2× 5 V LEDs**
1× Kingbright L-53GD
(5 mm green) and
1× Kingbright L-531D
(5 mm red) are
ideal and need no
limiting resistors
- ♦ **Breadboard and wire links**
- ♦ **5 V Power supply**
(only 20 mA current needed)

Figure 5 (Right) ♦
The OR gate output produces logic 1 when either input or both inputs are at logic 1, otherwise the output is at logic 0

Figure 6 (Far Right) ♦
The NAND gate output produces logic 1 when both inputs are at logic 0, otherwise the output is at logic 1



OR		
A	B	Out
0	0	0
0	1	1
1	0	1
1	1	1



NAND		
A	B	Out
0	0	1
0	1	1
1	0	1
1	1	0

QUICK TIP

Before you build the circuit shown in Figure 7, it's a good idea to build each of the individual gate circuits in Figures 4 to 6 separately to begin with, so you can test each circuit to make sure they give the results shown in their respective truth tables. If you fit the two input switches to the breadboard first, you can use these to test each of the gate circuits. You'll also need to include the two output LEDs. Building these three logic gate circuits separately will enable you to find the best layout for each gate circuit and help you to understand how each gate circuit works before attempting the complete half adder.

Notice in this circuit that the two transistors Tr5 and Tr6 are effectively connected in series; that is, the same current will be flowing through Tr5 (collector to emitter) and through Tr6 (collector to emitter). Therefore, for this current to develop a voltage across R12, both transistors must be conducting. All the transistors in the whole of the half adder circuit are operated in switch mode; that is, they will either be switched on, by a large enough current flowing into their base connection to cause the transistor to conduct heavily, or switched off by removing the base current and therefore preventing any collector/emitter current. The base current for Tr5 and Tr6 depends on the values of R9 and R8, which are both 6.8kΩ, and the switches Sw A and Sw B are used to simply connect R9 and R8 to the +5V supply.

So, when both transistors are switched on, the current through Tr5 and Tr6 will develop an output voltage across R12 (4.7kΩ) equal to the supply voltage, minus a small voltage drop due to the voltages across the PN junctions in Tr5 and Tr6.

As shown in Figure 3, one AND circuit will be used to drive the Sum output of the half adder and the other AND circuit will drive the Carry output.

THE OR GATE

As in the AND gate, the transistors here are being used as electronic switches and are activated by the currents flowing into their bases from Sw A and Sw B via the 6.8kΩ resistors R4 and R5 (Figure 5). However, in this circuit the transistors Tr3 and Tr4 are connected in parallel, with the collector of Tr3 connected directly to the collector of Tr4

and the emitters of both transistors also connected together. Therefore if either Tr3 or Tr4 is made to conduct, a voltage almost equal to the +5V supply will be developed across the output resistor R10. This therefore produces

“ The half adder does not carry out an impressive amount of arithmetic, but it only takes around 30 nanoseconds to produce its answer ”

what we shall call logic 1 at the output if Tr3, Tr4 or both transistors are made to conduct, fulfilling the requirements of the OR truth table and in the total half adder circuit, driving one of the inputs of the Sum AND gate as part of Figure 3's XOR function.

THE NAND GATE

Comparing the truth tables for the NAND gate in Figure 6 and the AND gate in Figure 4, it can be seen that the output columns of each table are the opposites of each other. This tells us that the circuit for the NAND gate will be similar to that of the AND

gate, but the outputs will have opposite logic values. Compare the circuits in **Figure 4** and **Figure 6** to see how this is done. The 4.7kΩ output resistor R12 connected from Tr6 emitter to ground in **Figure 4** is simply moved to become R1 in **Figure 6**. Now if both transistors in **Figure 6** are made to conduct at the same time, the voltage at the bottom of R1 will be almost 0V so producing logic 0 at the output; if either one or both transistors are switched off, however, the current path from R1 to ground will be interrupted and the output terminal will remain at logic 1, fulfilling the logic requirements of a NAND gate.

Figure 7 illustrates how two AND functions (shaded green), as well as the OR function (shaded blue) and NAND (shaded pink) logic functions needed for a half adder can be interconnected to drive two 5V LEDs representing the Sum and Carry outputs. The LEDs used in this project are designed for +5V supply circuits and have built-in current-limiting resistors so extra current-limiting resistors are not needed – unless you decide to use standard LEDs, in which case you will need to fit an appropriate current-limiting resistor in series with the LED.

The photograph (page 86) shows a practical layout of the half adder circuit on a dual breadboard. You can follow this layout to make a working half adder circuit or develop your own layout, depending on the breadboard and wire links you have available. Notice that only one half of the double board space has been used, so if you are feeling brave why not build a second half adder in the remaining board space and link the two circuits using another OR circuit, as shown in **Figure 2**, to make a full adder?

The half adder may not be as familiar as other more recognisable parts of a computer system such as disk drives, sound cards and touchscreens, but computers can work quite well without these, as proved by devices such as the Raspberry Pi and Arduino. However, no computer – not the Raspberry Pi, not the Arduino, nor even your pocket calculator – can work without the half adder, one of the unsung heroes of computing!

If studying adders in this article has inspired you to find out more about them, and how binary arithmetic really works to give the basic adders described here the power to carry out some awesome arithmetic, take a look at hsmag.cc/qnFyCp, where millions of people dedicated to learning about electronics go to study these fascinating combinational logic circuits essential to computing, as well as many more digital and analogue topics. □

Figure 7 ♦ This is the complete half adder circuit, combining Figures 4 to 6

